C^{0} -vector field, measure propagation and wave observability on a rough manifold

Jérôme Le Rousseau

LAGA Université de Paris 13

The celebrated Rauch-Taylor/Bardos-Lebeau-Rauch geometric control condition is central in the study of the observability of the wave equation linking this property to high-frequency propagation along geodesics that are therays of geometric optics. This connection is best understood through the propagation properties of microlocal defect measures that appear as solutions to the wave equation concentrate. If one considers a merely C^1 -metric, the vector fields that generates geodesics is C^0 . Thus, geodesics do exist but uniqueness is lost in general. Here, first on a compact manifold without boundary, and second on a manifold with boundary, in the case of the wave equation with homogeneous Dirichlet conditions, we consider this low regularity setting, revisit the geometric control condition, and address the question of support propagation for a measure solution to an ODE with continuous coefficients. This leads to a sufficient condition for the observability and equivalently the exact controllability of the wave equation.